Answers

Bonn Integration Bee

May 18, 2025

This file contains answers for the integrals.

Kahoot

1.
$$\int 2025BIBee\,\mathrm{d}e = 675BIBeee$$

2.
$$\int_0^{2025} \lfloor x \rfloor\,\mathrm{d}x = 2049300$$

3.
$$\int_0^{\frac{\pi}{2}} \sin x \sin 2x\,\mathrm{d}x = \frac{2}{3}$$

4.
$$\int_0^{\pi/2} \frac{\cos(x)}{1+\sin(x)}\mathrm{d}x = \ln(2)$$

5.
$$\int_{-\pi}^{\pi} \lfloor \sin(\lfloor x \rfloor) \rfloor\,\mathrm{d}x = -3$$

6.
$$\int_0^{\pi} \sin^{2025}(x)\cos^{2025}(x)\mathrm{d}x = 0$$

7.
$$\int_0^1 \lfloor 4x(1-x) \rfloor\,\mathrm{d}x = 0$$

8.
$$\int_0^1 \mathrm{d}x_1 \int_0^{x_1} \mathrm{d}x_2 \cdots \int_0^{x_{2024}} \mathrm{d}x_{2025} = \frac{1}{2025!}$$

9.
$$\int_1^e \frac{\cos(1+\ln(x))}{x}\,\mathrm{d}x = \sin(2) - \sin(1)$$

10.

$$\int_0^{\frac{\pi}{2}} \min(\sin x, \cos x) \max(\tan x, 1)\, \mathrm{d}x = 1$$

11.

$$\int_0^1 \frac{\arcsin(x)}{\sqrt{1-x^2}} \mathrm{d}x = \frac{\pi^2}{8}$$

12.

$$\int_{\frac{1}{e}}^1 x^{\frac{1}{\ln(x)}}\, \mathrm{d}x = e - 1$$

13.

$$\int_0^{4\pi} \sin(\sin x - x) \mathrm{d}x = 0$$

14.

$$\int_0^\infty \frac{x^{1011}}{1+x^{2024}}\, \mathrm{d}x = \frac{\pi}{2024}$$

15.

$$\int_{-\infty}^0 e^{e^x + x}\, \mathrm{d}x = e - 1$$

16.

$$\int_0^\infty \frac{e^{-x}}{e^{-x} + 2025}\, \mathrm{d}x = \ln\left(\frac{2026}{2025}\right)$$

17.

$$\int_0^1 45x^{45} \mathrm{d}x = \frac{45}{46}$$

18.

$$\int_0^e \ln\left(x^2\right) \mathrm{d}x = 0$$

19.

$$\lim_{N\to\infty} \int_0^1 \sum_{i=1}^N \frac{1}{i} x^{i-1} \mathrm{d}x = \frac{\pi^2}{6}$$

20.

$$\int_{\frac{1}{2025}}^{2025} e^{x+\frac{1}{x}} \frac{\log_{2025} x}{x} \mathrm{d}x = 0$$

21.

$$\int_0^2 1 + 2x + 3x^2 + 4x^3 + 5x^4 + 6x^5\, \mathrm{d}x = 126$$

22.

$$\int_1^e \frac{\ln(\ln x + 1)}{x} \mathrm{d}x = 2\log 2 - 1$$

23.

$$\int_0^1 \frac{\sqrt{x}}{2(1+x)}\,\mathrm{d}x = 1 - \frac{\pi}{4}$$

24.

$$\int_0^{\frac{\pi}{2}} \cot x - x\csc^2 x\,\mathrm{d}x = -1$$

25.

$$\int_0^{\frac{\pi}{2}} (x^4 - 4\pi x^3 + 6\pi^2 x^2 - 4\pi^3 x + \pi^4)\,\mathrm{d}x = \frac{\pi^5}{155}$$

Quarterfinals

First leg

1.

$$\int \tan^3(\sin x)\cos x \mathrm{d}x = \log\cos\sin x + \frac{1}{2\cos^2(\sin x)}$$

2.

$$\int_1^{\infty} \frac{1}{x^2+x+1} + \frac{1}{x^3+x^2+x} + \frac{1}{x^4+x^3+x^2}\,\mathrm{d}x = 1$$

3.

$$\int_0^{\infty} 2x^2 e^{-x^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2}$$

4.

$$\int_0^{\frac{3}{4}} \mathrm{arsinh} x\,\mathrm{d}x = \frac{3\log 2 - 1}{4}$$

5. Tiebreaker 1

$$\int_0^1 \ln\left(1+x^2\right)\mathrm{d}x = \ln 2 - 2 + \frac{\pi}{2}$$

6. Tiebreaker 2

$$\int_0^1 (1-x)(1+x+x^2+x^3+x^4+x^6+x^7)\,\mathrm{d}x = \frac{109}{126}$$

Second leg

1.

$$\int_0^{\pi} \sin^2 x\cos^2 x \mathrm{d}x = \frac{\pi}{8}$$

2.

$$\int_0^{\frac{\pi}{2}} \sin x\sinh x \mathrm{d}x = \frac{1}{2}\cosh\frac{\pi}{2}$$

3.
$$\int_{e^2}^{e^4} \frac{1}{x \log_2(x) \ln \ln x} \, \mathrm{d}x = (\ln 2)^2$$

4.
$$\int_0^1 x^{2025} (\ln x)^{2025} \, \mathrm{d}x = -\frac{2025!}{2026^{2026}}$$

5. Tiebreaker 1
$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1 + \sin^2 x} \, \mathrm{d}x = \frac{\log\left(\sqrt{2} + 1\right)}{\sqrt{2}}$$

6. Tiebreaker 2
$$\int x^{100000000000000000000} \, \mathrm{d}x = \frac{1}{100000000000000000001} x^{100000000000000000001}$$

Semifinals

1.
$$\int_0^{\frac{\pi}{3}} \frac{1}{1 - \sin x} \, \mathrm{d}x = \sqrt{3} + 1$$

2.
$$\int_0^{101} \frac{\lfloor x \rfloor}{1 + 2\lfloor x \rfloor x - \lfloor x \rfloor (\lfloor x \rfloor + 1)} \, \mathrm{d}x = \frac{\log 10101}{2}$$

3.
$$\int \frac{\cos(\ln x) - \sin(\ln x)}{x^2} \, \mathrm{d}x = \frac{\sin(\ln x)}{x}$$

4.
$$\int_0^2 \lfloor \ln x \rfloor \, \mathrm{d}x = -\frac{e}{e - 1}$$

5. Tiebreaker 1
$$\int_0^1 \arctan(x) \, \mathrm{d}x = \frac{\pi}{4} - \frac{\log 2}{2}$$

6. Tiebreaker 2
$$\int_0^\infty e^{-x} \cos x \, \mathrm{d}x = \frac{1}{2}$$

Third Place Playoff

1. $$\int_0^\infty \frac{1+x^2}{1+x^4}\,\mathrm{d}x = \frac{\pi}{\sqrt{2}}$$

2. $$\int_0^{\frac{3}{4}} 2x \operatorname{arsinh} x + \sqrt{x^2+1}\mathrm{d}x = \frac{25}{16}\log 2$$

3. $$\int_{-\log 2}^0 \sqrt{1-e^x}\,\mathrm{d}x = 2\log\left(\sqrt{2}+1\right) - \sqrt{2}$$

4. $$\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + i\cos x}\,\mathrm{d}x = 1 - i$$

5. Tiebreaker 1
$$\int_{e^2}^{e^4} \log_x(2)\log_{\log x}(4)\log_{e^x}(8)\,\mathrm{d}x = 6(\log 2)^4$$

6. Tiebreaker 2
$$\int_0^1 \min(2x^2-1, 1-x^2)\,\mathrm{d}x = \frac{2}{3} - \frac{4\sqrt{6}}{9}$$

Epic Finale

1. $$\int_0^1 \ln\left(1+\sqrt[3]{x}\right)\mathrm{d}x = 2\ln 2 - \frac{5}{6}$$

2. $$\int_{\frac{1}{2}}^1 \frac{1}{x^2}\sqrt[3]{\frac{x}{1-x}}\,\mathrm{d}x = 3 + 2\sqrt{3}$$

3. $$\int_0^1 \frac{\ln x}{1-x}\,\mathrm{d}x = -\frac{\pi^2}{6}$$

4. $$\int_0^1 \frac{x^2}{\sqrt{x^2+3}}\,\mathrm{d}x = 1 - \frac{3}{2}\operatorname{arsinh}\frac{1}{\sqrt{3}}$$

5. Tiebreaker 1
$$\int_0^{\frac{\pi}{2}} \sin(51x)\cos(24x)\,\mathrm{d}x = \frac{51}{2025}$$

6. Tiebreaker 2
$$\int_2^3 \frac{x^2+2026x+2025}{x^2+2024x-2025}\,\mathrm{d}x = 1 + 2\log 2$$

Reserves for the case of the integrals being too easy

This just won't happpen.

1.
$$\int_0^{2\pi} \log\left(\frac{5}{4} + \cos(t)\right) \mathrm{d}t = 0$$

Removed Integrals

1.
$$\int_0^1 \sqrt{\frac{\sqrt{x^5}}{1-\sqrt{x^3}}}\,\mathrm{d}x = \frac{\pi}{3}$$

2.
$$\int_0^{\frac{1}{2}} \sqrt{1-x^2}\mathrm{d}x = \frac{\sqrt{3}}{8} + \frac{\pi}{12}$$