Bibee 2026 Integrals WITH SOLUTIONS

Integral selection committee

May 2026

1 Kahoot

1. **Series-ly???**
   $$\int_0^{\pi} \sum_{k=1}^{2026} \frac{1+\cos(2kx)}{2}\,\mathrm{d}x$$
   Solve by switching sum and integral. $\forall k \in \mathbb{Z} : \int_0^{\pi} \cos(2kx)\,\mathrm{d}x = 0$ and $\int_0^{\pi} \frac{1}{2}\,\mathrm{d}x = \frac{\pi}{2}$. We get $\boxed{1013\pi}$ as our solution.
2. **I know this one!**
   $$\int_{-\infty}^{0} x^2 e^{x^3}\,\mathrm{d}x$$
   Substitute $u = x^3$. The solution is $\boxed{\frac{1}{3}}$.
3. **Watch out!**
   $$\int_{-\infty}^{\infty} e^{-z^2}\,\mathrm{d}x$$
   This is NOT a Gaussian integral because we are integrating with respect to z instead of x. The right solution is $\boxed{\infty}$.
4. **Watch out! (Part 2)**
   $$\int_{-\infty}^{\infty} \lfloor e^{-\lceil x^2 \rceil} \rfloor \mathrm{d}x$$
   $= \boxed{0}$
5. **Trig-Trag-Troe.**
   $$\int \frac{\mathrm{d}x}{\sec(x) + \tan(x)\sin(x)}$$
   We multiply top and bottom by $\cos(x)$, substitute $u = \sin(x)$ and get $\boxed{\tan^{-1}(\sin(x))}$.
6. **Determinated.**
   $$\int_0^{\frac{1}{3}} 3\det\begin{pmatrix} 1 & e^{-x} & e^{x} \\ e^{x} & 1 & e^{-x} \\ e^{-x} & e^{x} & 1 \end{pmatrix} \mathrm{d}x$$

Sarrus gives us the determinant: $e^{-3x} + e^{3x} - 2$. We integrate and get $\boxed{e - \frac{1}{e} - 2}$.

7. **Funny?**
$\int_6^7 (6x^2 - 7x^2)^{6-7}\,\mathrm{d}x$
$= -\int_6^7 \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{6} + \frac{1}{7} = \boxed{-\frac{1}{42}}$

8. **Yes, you have to actually do this**
$\int_0^{24} \lfloor \sin x \rfloor \mathrm{d}x$
Split up the integral at π, 2π, 3π, 4π, 5π, 6π, 7π to get $0 - \pi + 0 - \pi + 0 - \pi + 0 - (24 - 7\pi) = \boxed{4\pi - 24}$

9. **Beep beep**
$\int_0^{\pi} \frac{1}{1+\tan x} + \frac{1}{1+\cot x}\mathrm{d}x$
$= \int_0^{\pi} \frac{\cos x}{\sin x + \cos x} + \frac{\sin x}{\sin x + \cos x}\mathrm{d}x = \boxed{\pi}$

10. **Can you FIX it?**
$\int_0^1 [\cos(\arctan(x)) + \sin(\arctan(x))]^2 + [\cos(\arctan(x)) - \sin(\arctan(x))]^2 \mathrm{d}x$
$= \boxed{2}$. Proof by obviousness.

11. **How fast am I, going to infinity?**
$\int_0^x \cos(\cos(\cos(\cos(\cos(...(t)...)))))\mathrm{d}t$
Since infinite cosines are leaving us with a constant (tryhards use Banach's Fix Point Theorem on this), it is $\boxed{\Theta(x)}$.

12. **Dividing Polynomials**
$\int_0^1 \frac{x^3 + 3x^2 + 3x}{x+1}\mathrm{d}x$
$= \int_0^1 \frac{(x+1)^3 - 1}{x+1}\mathrm{d}x = \int_1^2 \frac{y^3 - 1}{y}\mathrm{d}y = \boxed{\frac{7}{3} - \ln(2)}$

13. **What's our favorite animal?**
$\int_0^2 \int_0^2 \int_0^2 \int_0^2 \int_0^2 beeee\,\mathrm{d}b\,\mathrm{d}e\,\mathrm{d}e\,\mathrm{d}e\,\mathrm{d}e$
$= \int_0^2 \int_0^2 \int_0^2 \int_0^2 2e^4\,\mathrm{d}e\,\mathrm{d}e\,\mathrm{d}e\,\mathrm{d}e = \int_0^2 \int_0^2 \int_0^2 \frac{64}{5}\,\mathrm{d}e\,\mathrm{d}e\,\mathrm{d}e = \frac{64 \cdot 2^3}{5} = \boxed{\frac{512}{5}}$.

14. **The world's smallest violin!**

$$\int_0^{0.2026} \frac{1}{2\sqrt{|𝄞-0.0001|}}\,\mathrm{d}\,𝄞$$

This equals $\operatorname{sgn}(𝄞)\sqrt{|𝄞|}\Big|_{-0.0001}^{0.2025} = 0.45 + 0.01 = \boxed{0.46}$.

15. **Another 2026!**

$$\int_0^{\sqrt{2026}} \frac{(2x)^2}{\sqrt{2026-x^2}}\mathrm{d}x$$

$$= 4\int_0^{\pi/2} 2026\sin^2(u)\mathrm{d}u = 4\ 2026\left(\frac{u}{2}-\frac{\sin(2u)}{4}\right)\Big|_0^{\pi/2} = \boxed{2026\pi}$$

substituting $x = \sqrt{2026}\sin(u)$ and using your favorite way of integrating $\sin^2$.

16. **Weeeeeeeeeeeeeeeee!**

$$\int_0^1 e^{e^{e^{e^{e^x}}}}\, e^{e^{e^{e^x}}}\, e^{e^{e^x}}\, e^{e^x}\, e^x\,\mathrm{d}x$$

$$= \left[e^{e^{e^{e^{e^x}}}}\right]_{x=0}^1 = \boxed{e^{e^{e^{e^{e}}}} - e^{e^{e^{e}}}}.$$

17. **Trippy greeting from the MIT :O**

$$\int_{\int_{\int_{\int_0^1 x\,\mathrm{d}x}^1 x\,\mathrm{d}x}^1 x\,\mathrm{d}x}^{\int_0^{\int_0^{\int_0^1 x\,\mathrm{d}x} x\,\mathrm{d}x} x\mathrm{d}x} x\,\mathrm{d}x = \int_{\frac{55}{128}}^{\frac{1}{128}} x\,\mathrm{d}x = \boxed{-\frac{189}{2048}}.$$

Remark: This is stolen from the MIT Integration Bee 2026.

18. **A radical product.**

$$\int_0^1 \prod_{n=1}^{\infty} \sqrt[n^2]{1+x}\,\mathrm{d}x$$

Use that $\prod_{n=1}^{\infty}(1+x)^{\frac{1}{n^2}} = (1+x)^{\sum_{n=1}^{\infty}\frac{1}{n^2}} = (1+x)^{\frac{\pi^2}{6}}$. The solution $\boxed{\dfrac{2^{\frac{\pi^2}{6}+1}-1}{\frac{\pi^2}{6}+1}}$ follows directly.

19. **Is it even integrable?**

$$\int x^{\frac{\ln\sin x}{\ln x}}\mathrm{d}x$$

$$= \int \sin x\mathrm{d}x = \boxed{-\cos x}.$$

20. **A famous one**
$$\int_1^\infty \frac{1}{\lceil x \rceil^2} \mathrm{d}x$$
$$= \boxed{\frac{\pi^2}{6} - 1}$$

21. **Sneaky Euler.**
$$\int_0^{\frac{\pi}{2}} e^{ix} \cos(x) \, \mathrm{d}x$$
We use $\cos(x) = \frac{e^{ix}+e^{-ix}}{2}$, simplify via algebraic manipulation and Euler's identity. Solution: $\boxed{\frac{\pi}{4} + \frac{i}{2}}$.

22. **Back to the roots!**
$$\int_0^5 \sqrt{x\sqrt{x\sqrt{x\ldots}}} \, \mathrm{d}x$$
By the identity: $x^{1/2+1/4+\cdots} = x^1$ we get above is $\displaystyle\int_0^5 x = 12.5$

23. **Log salad.**
$$\int_{e^2}^{e^4} \frac{\ln 2x}{x \ln x} \mathrm{d}x$$
$$= \int_{e^2}^{e^4} \frac{\ln x + \ln 2}{x \ln x} \mathrm{d}x = \ln x + \ln 2 \ln \ln x|_{e^2}^{e^4} = \boxed{2 + \ln^2 2}$$

24. **OG change of basis**
$$\int_0^1 5^{\log_2 x} \mathrm{d}x$$
$$= \int_0^1 x^{\frac{1}{\log_5 2}} \mathrm{d}x = \frac{1}{1 + \frac{1}{\log_5 2}} = \boxed{\log_{10} 2}$$

25. **Welcome to Bonn Integration Bee**
$$\int_0^\infty 2026^{-x} \mathrm{d}x$$
$$= \frac{1}{\ln 2026}$$

2 Tiebreakers

1. **QuadWave!**
   $$\int \sin^2(x)\cos^2(x)\mathrm{d}x$$
   $$= \frac{1}{4}\int \sin^2(2x)\mathrm{d}x = \frac{1}{8}\int \sin^2(u)\mathrm{d}u = \boxed{\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}}$$
   Mostly irrelevant comment by Leandro: This is equivalent to $\frac{x}{8} - \frac{\sin(4x)}{32}$.

2. **I hope you have good eyes.**
   $$\int_0^1 2026202620262026202620262026^x \mathrm{d}x =$$
   $$\boxed{\frac{(2026202620262026202620262025)}{\ln(2026202620262026202620262026)}}$$

3. **Geometry... without triangles!?**
   $$\int \frac{(1+x+x^2+\cdots+x^{2024})(1-x^3)}{1+x+x^2}\,\mathrm{d}x$$
   The integrand simplifies to $1 - x^{2025}$, so the answer is $\boxed{x - \frac{1}{2026}x^{2026}}$

4. **Is this what you think?**
   $$\int_0^3 4\lfloor x\rfloor^{\lceil x\rceil^{\lfloor x\rfloor}} + \lceil x\rceil^{\lceil x\rceil} - \lfloor x\rfloor^{\lfloor x\rfloor}\mathrm{d}x$$
   $= \boxed{2080}$
   This is not $= 2026$ because we f****d up a little.

5. **Neper's Root**
   $$\int \sqrt{e^x - 1}\,\mathrm{d}x$$
   Substitute $t = \sqrt{e^x - 1}$ to get $\int \frac{2t^2}{1+t^2}dt$, then standard techniques lead to $\boxed{2\sqrt{e^x-1} - 2\arctan\sqrt{e^x-1}}$.

6. **See you on the 3rd floor.**
   $$\int_0^3 2(\lfloor x\rfloor^{\lceil x\rceil^{\lfloor x\rfloor}} + 250\lfloor\sqrt{x}\rfloor)\,\mathrm{d}x$$
   We evaluate piecewise and get $2(0+1+512) + 500(2) = \boxed{2026}$

7. **There is always one trig integral more.**
   $$\int \frac{\mathrm{d}x}{9\cos^2(x) + 4\sin^2(x)}$$
   Use the substitution $t = \tan x$ to get $\int \frac{\mathrm{d}t}{9+4t^2}$, so the solution is
   $$\boxed{\frac{1}{6}\arctan\left(\frac{2\tan(x)}{3}\right)}$$

8. **Double root.**
$$\int_0^1 \sqrt{x}e^{\sqrt{x}}\mathrm{d}x$$
$$= 2\int_0^1 u^2 e^u \mathrm{d}u = 2(u^2 - 2u + 2)e^u\Big|_0^1 = 2e - 4$$

9. **From floor to ceiling.**
$$\int_0^5 \left(\left\lfloor \frac{x}{2} \right\rfloor - \left\lceil \frac{x}{3} \right\rceil\right) \mathrm{d}x$$
$$= \int_0^2 \mathrm{d}x + \int_2^3 2\,\mathrm{d}x + \int_3^4 \mathrm{d}x + \int_4^5 2\,\mathrm{d}x = \boxed{7}$$

10. **Where does π come from?**
$$\int_{1/e}^{e} \arctan x \frac{(\ln x)^{2026}}{x}\, dx$$
Call the integral I, use the substitution $t = 1/x$ to see $I = \int_{1/e}^{e} \arctan \frac{1}{x} \frac{(\ln x)^{2026}}{x}\, dx$, then sum up using $\arctan x + \arctan \frac{1}{x} = \frac{\pi}{2}$ so that $2I = \frac{\pi}{2}\int_{1/e}^{e} \frac{(\ln x)^{2026}}{x}\, dx = \frac{\pi}{2}[\frac{(\ln x)^{2027}}{2027}]_{1/e}^{e} = \frac{\pi}{2027}$. Finally $\boxed{I = \frac{\pi}{4054}}$

11. **Happy New Year!?**
$$\lim_{R\to\infty}\int_{-R}^{R}\prod_{j=0}^{2026}\frac{1}{j+1}x^{j^2+j+1}\,\mathrm{d}x$$
$j^2 + j + 1 = j(j+1) + 1$ is odd. Hence, the integral collapses to $\boxed{0}$.

12. **The last integral of the evening is really shit.**
$$\int_0^1 x^{13} + 67x + x^9 + 13x^{12} + 3x^6 + 2(x+1)^{-3}\mathrm{d}x$$
$$= \boxed{\frac{5019}{140}}$$

3 QF

1. **Root of all Problems.**
$$\int_1^2 \frac{x^5}{\sqrt{x^3-1}}\mathrm{d}x$$
$$= \frac{1}{3}\int_1^8 \frac{u}{\sqrt{u-1}}\mathrm{d}u = \frac{1}{3}\int_1^8 \sqrt{u-1} + \frac{1}{\sqrt{u-1}}\mathrm{d}u = \boxed{\frac{20\sqrt{7}}{9}}$$

2. **Antagonist views**
$$\int \frac{4x^2+6x+1}{x^3+2x^2+x+2}\mathrm{d}x$$
$$= \int \frac{1}{x+2} + \frac{3x}{x^2+1}\mathrm{d}x = \boxed{\ln|x+2| + \frac{3}{2}\ln\left(x^2+1\right)}$$

3. **Walking in circles.**
$$\int_0^{2\pi} \prod_{n=0}^{2026}(n^2+1)\cos\left(\frac{x}{3}\prod_{n=0}^{2026}(n^2+1)\right)\mathrm{d}x$$
Write $A = \prod_{n=0}^{2026}(n^2+1)$. The integral is $\int_0^{2\pi} A\cos\left(\frac{x}{3}A\right)\mathrm{d}x$ or $3\int_0^{2\pi A/3}\cos x\mathrm{d}x$. It can be computed that A is an integer 2 modulo 3. By periodicity, the answer is $3(\sin\left(\frac{4\pi}{3}\right) - \sin(0)) = \boxed{-\frac{3\sqrt{3}}{2}}$.

4. **Fraction Action!?**
$$\int_0^\infty \frac{\{x\}}{(\lfloor x\rfloor+1)^2}\,\mathrm{d}x$$
Write the integral as $\sum_{n=0}^{\infty}\int_n^{n+1}\frac{x-n}{(n+1)^2}\,\mathrm{d}x = \sum_{n=0}^{\infty}\frac{1}{(n+1)^2}\int_0^1 u\,du$
$$= \frac{1}{2}\sum_{n=0}^{\infty}\frac{1}{(n+1)^2} = \frac{1}{2}\cdot\frac{\pi^2}{6} = \boxed{\frac{\pi^2}{12}}$$

5. **Unreasonably rational!**
$$\int \frac{2x^2+3x+1}{x^3+x^2+x}\,\mathrm{d}x$$
Solve with partial fractions: $\frac{2x^2+3x+1}{x^3+x^2+x} = \frac{1}{x} + \frac{x+2}{x^2+x+1}$.
Note that $x+2 = \frac{1}{2}(2x+1)+\frac{3}{2}$. To evaluate $\int \frac{1}{x^2+x+1}\,\mathrm{d}x$, we complete the square and calculate $\int \frac{1}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\,\mathrm{d}x = \frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)$.
The solution is given by $\boxed{\ln|x| + \frac{1}{2}\ln\left(x^2+x+1\right) + \sqrt{3}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)}$

6. **Digitization!**
Let $s(n)$ denote the **digit sum of n**. / Sei $s(n)$ die **Quersumme von n**.

$$\int_0^1 s(\lfloor 100x \rfloor)\mathrm{d}x$$

Because of linearity, the integral equals $\displaystyle\sum_{k=0}^{99} \int_{\frac{k}{100}}^{\frac{k+1}{100}} s(k)\mathrm{d}x = \frac{1}{100}\sum_{k=0}^{99} s(k)$.
Let $k = 10a + b, \quad a, b \in \{0, ..., 9\}$ such that $s(n) = a + b$.
It follows: $\displaystyle\sum_{k=0}^{99} s(n) = \sum_{a=0}^{9}\sum_{b=0}^{9}(a+b) = 10\sum_{a=0}^{9} a + 10\sum_{b=0}^{9} b = 900$.
So the solution is $\frac{900}{100} = \boxed{9}$.

7. **Dotsy and messy**

$$\int_0^1 \sqrt{x^2 + x + \sqrt{x^2 + x + \sqrt{\ldots}}}\,dx$$

Calling the integrand y we have by recursion $y = \sqrt{x^2 + x + y}$, so that $y^2 - y = x^2 + x$, hence $y - \frac{1}{2} = \pm(x + \frac{1}{2})$. By positivity $y = x + 1$ so we are left with $\int_0^1 (x+1)dx$ and the answer is $\boxed{\frac{3}{2}}$

8. **Some ancient Greek would be proud for sure...**

$$\int_0^1 \arctan\left(\frac{\sqrt{1-x^2}}{x}\right)\mathrm{d}x$$

substitute $x = \cos(y)$ and get $\boxed{1}$ or do some really shitty workaround using Thales' theorem like Leandro xD

4 SF

1. **If you know, you know.**
$$\int_0^{\pi/4} \frac{\sqrt{\tan(x)}}{\cos^4(x)} \mathrm{d}x$$
$$= \int_0^1 \frac{\sqrt{u}}{\cos^2(\arctan(u))} \mathrm{d}u = \int u^{5/2} + u^{1/2} \mathrm{d}u = \left[\frac{2}{7}u^{7/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \boxed{\frac{2}{7} + \frac{2}{3}}$$
substituting $u = \tan x$ and using $\cos(\arctan(u)) = \frac{1}{\sqrt{u^2+1}}$.

2. **Polynomial Chaos!**
$$\int_{\ln\left(\frac{e^2}{\sqrt{5}}\right)}^{\ln\left(e^2\sqrt{5}\right)} (x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 30)(\cos(x)\cos(2) + \sin(x)\sin(2))\, \mathrm{d}x$$
$$= \int_{\ln\left(\frac{e^2}{\sqrt{5}}\right)}^{\ln\left(e^2\sqrt{5}\right)} (x-2)^5 \cos(x-2)\, \mathrm{d}x + 2\int_{\ln\left(\frac{e^2}{\sqrt{5}}\right)}^{\ln\left(e^2\sqrt{5}\right)} \cos(x-2) \mathrm{d}x = \boxed{4\sin\left(\ln\left(\sqrt{5}\right)\right)}.$$
The first is just $= \boxed{0}$ by symmetry around $x = 2$.

3. **Delicious!!**
$$\lim_{n\to\infty} \int_1^9 \sum_{k=1}^{n} \frac{1}{\sqrt{n^2x + k}} \,\mathrm{d}x$$
Write $a_n = \sum_{k=1}^n \frac{1}{\sqrt{n^2x+k}}$. First, we exchange limit and integral and compute $\lim_{n\to\infty} a_n = \frac{1}{\sqrt{x}}$. This follows from $a_n \geq n \cdot \frac{1}{\sqrt{n^2x+n}} = \frac{1}{\sqrt{x+\frac{1}{n}}}$ and $a_n \leq n \cdot \frac{1}{\sqrt{n^2x}} = \frac{1}{\sqrt{x}}$ by applying the Sandwich-lemma. Hence the integral is $\displaystyle\int_1^9 \frac{1}{\sqrt{x}} \,\mathrm{d}x = [2\sqrt{x}]_1^9 = 6 - 2 = \boxed{4}$.

4. $\pi/3$ **and** $\pi/4$ **would be worse boundaries**
$$\int_0^{\pi/3} \tan^2(x) x \mathrm{d}x$$
$$= (\tan x - x)x - \int_0^{\pi/3} \tan x - x \mathrm{d}x = \left[x\tan x + \ln(\cos(x)) - \frac{x^2}{2}\right]_0^{\pi/3} =$$
$\boxed{\frac{\pi}{\sqrt{3}} - \ln 2 - \frac{\pi^2}{18}}$, where $\int \tan^2(x)\mathrm{d}x = \int \tan^2(x) + 1 - 1\mathrm{d}x = \tan x - x$.

5 3rd Place

1. **This Partialbruchzerlegung is nasty.**
   $$\int_{\sqrt{2}}^{\sqrt{3}} \frac{x^3(x^2+3)}{(x^4-1)(x^2+1)}\mathrm{d}x$$
   $$= \int_{\sqrt{2}}^{\sqrt{3}} \frac{x^5+3x^3}{x^6+x^4-x^2-1}\mathrm{d}x = \frac{1}{2}\int_2^3 \frac{u^2+3u}{u^3+u^2-u-1}\mathrm{d}u = \frac{1}{2}\int_2^3 \frac{1}{(u+1)^2}+ \frac{1}{u-1}\mathrm{d}u = \boxed{\frac{1}{24}+\frac{\ln(2)}{2}}$$
   The trick is to see, that you can substitute $u = x^2$, then the partial fractions calculation becomes easy.

2. **Modest but beautiful.**
   $$\int \frac{\sin(x)}{\sin(x)+1}\mathrm{d}x$$
   $$= \int 1 - \frac{1}{\sin(x)+1}\mathrm{d}x = x - \int \frac{1-\sin(x)}{\cos^2(x)}\mathrm{d}x = \boxed{x - \tan(x) + \sec(x)}$$

3. **Tricky Trigs.**
   $$\int_0^{\frac{\pi}{2}} (\sin(x)^{2\cos(x)-1}\cos^2(x) - \sin(x)^{2\cos(x)+1}\ln(\sin(x))\,\mathrm{d}x$$
   Basic algebraic manipulation gives us
   $$\int_0^{\pi} \sin(x)^{\cos(x)}[\sin(x)^{\cos(x)}(\frac{\cos^2(x)}{\sin(x)} - \sin(x)\ln(\sin(x)))]\,\mathrm{d}x,$$
   substitute $u = \sin(x)^{\cos(x)}$ since $u' = \sin(x)^{\cos(x)}\frac{\cos^2(x)}{\sin(x)} - \sin(x)\ln(\sin(x))$.

   We evaluate and get $\boxed{\frac{1}{2}}$ as our solution.

   This is exactly the derivative of $\sin^{2\cos}$.

4. **Quotients**
   $$\int_1^{\infty} \frac{e^{-x}}{x} + \frac{e^{-x}}{x^2}\mathrm{d}x$$
   $$= \int_1^{\infty} \frac{e^{-x}}{x}\mathrm{d}x + \left(-\frac{e^{-x}}{x}\Big|_1^{\infty} - \int_1^{\infty} \frac{e^{-x}}{x}\mathrm{d}x \right) = -\frac{e^{-x}}{x}\Big|_1^{\infty} = \boxed{e^{-1}}$$
   You also could just invert the quotient rule immediately.

6 Final

1. **Roots bloody roots!**
$$\int_0^\infty \frac{\ln x}{(1+x^{\sqrt{2}})^{\sqrt{2}}}\,\mathrm{d}x$$
Just use $t = x^{-1}$ and symmetry to get $\boxed{0}$, but it's cool to see :)

2. **Can you spot the ODE?**
$$\int_{-\infty}^0 \frac{(x+\sqrt{1+x^2})^{2026}}{\sqrt{1+x^2}}\mathrm{d}x$$
$= \int_0^1 u^{2025}\mathrm{d}u = \boxed{\frac{1}{2026}}$, substituting $u = x + \sqrt{1+x^2}$ and using, that $\frac{d}{dx}x + \sqrt{1+x^2} = \frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}}$.

3. **Pure Purgatory.**
$$\lim_{n\to\infty} \sqrt{\pi n}\int_0^1 \frac{x^{2n}}{\sqrt{1-x^2}}\mathrm{d}x$$
We substitute $x = \sin(t)$ and get $\lim_{n\to\infty} \sqrt{\pi n}\int_0^{\frac{\pi}{2}} \sin^{2n}(x)\mathrm{d}x$. This can be solved with Wallis' Product and we get $\lim_{n\to\infty} \sqrt{\pi n}\frac{\pi}{2}\frac{(2n-1)!!}{(2n)!!}$. We either know by heart - or show by using the Stirling formula - that $\frac{(2n-1)!!}{(2n)!!}$ is asymptotically equivalent to $\frac{1}{\sqrt{\pi n}}$ and thus, the integral resolves to $\boxed{\frac{\pi}{2}}$.

4. **Not spherical, that's hyperbolic!**
$$\int \frac{\sqrt{9\cosh^2(x)\sinh^2(x) - 3\sinh^2(x) + 1}}{\cosh(x)\sinh(x)}\mathrm{d}x$$
$$= \int \frac{\sqrt{9(\sinh^2(x)+1)\sinh^2(x) - 3\sinh^2(x) + 1}}{(\sinh^2(x)+1)\sinh(x)}\cosh x\mathrm{d}x$$
$$= \int \frac{\sqrt{9u^4+6u^2+1}}{u^3+u}\mathrm{d}u = \int \frac{3u^2+1}{u^3+u}\mathrm{d}u = \ln\big(u^3+u\big) = \boxed{\ln\big(\sinh(x)\cosh^2(x)\big)}$$
substituting $u = \sinh(x)$ and using $\cosh^2 = \sinh^2 + 1$.

5. **Dancing constants**
$$\int_0^{\frac{\pi}{2}} \frac{e^{|x-\frac{\pi}{4}|}}{1+\tan^\varphi x}\mathrm{d}x.$$
Call the integral I, then by using $t = \frac{\pi}{2} - x$ one finds $I = \int_0^{\frac{\pi}{2}} \frac{e^{|x-\frac{\pi}{4}|}}{1+\tan^\varphi x}\tan^\varphi x\, dx$ so that $2I = \int_0^{\frac{\pi}{2}} e^{|x-\frac{\pi}{4}|}dx$ then split $\int_0^{\frac{\pi}{4}} + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ and conclude $\boxed{e^{\frac{\pi}{4}} - 1}$.

7 Bonus for those who want real stress in their life

1. **A Phi-nomenal Integral.**
   $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{(1+x^\phi)^\phi}$

   Substitute $u = x^\phi \to \frac{1}{\phi}\displaystyle\int_0^\infty \frac{u^{\frac{1}{\phi-1}}}{(1+u)^\phi}\,du$. This resolves into a β-function. We get $\frac{\Gamma(\frac{1}{\phi})}{\phi\Gamma(\phi)}$. Since $\phi\Gamma(\phi) \overset{\text{Gamma identity}}{=} \Gamma(\phi+1) \overset{\text{Golden ratio property}}{=} \Gamma(\frac{1}{\phi})$, the solution is $\boxed{1}$.